Machines

Division B/C

Georgia Tech Event Workshop Series
2025-26

Science Olympiad at Georgia Tech®

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Event Overview

- Hybrid (Test + Build Performance)
- Exam mostly on Mechanics (Physics)
- Build to find ratio of masses as fast as possible
- Different by division
  - Div B:
    - Class 1 lever,
    - Max Mass Ratio: 8-12,
    - 80cm Beam
  - Div C:
    - Compound machine,
    - Max mass ratio: 4-7
    - 40cm Beam

MACHINES C

See General Rules, Eye Protection & other Policies on www.soinc.org as they apply to every event.

1. **DESCRIPTION:** Teams will construct a lever-based measuring device prior to the tournament to determine the mass ratios between three test masses and complete a written test on simple and compound machine concepts.

   A TEAM OF UP TO: 2 **APPROXIMATE TIME:** 50 minutes

   CALCULATOR: Class III

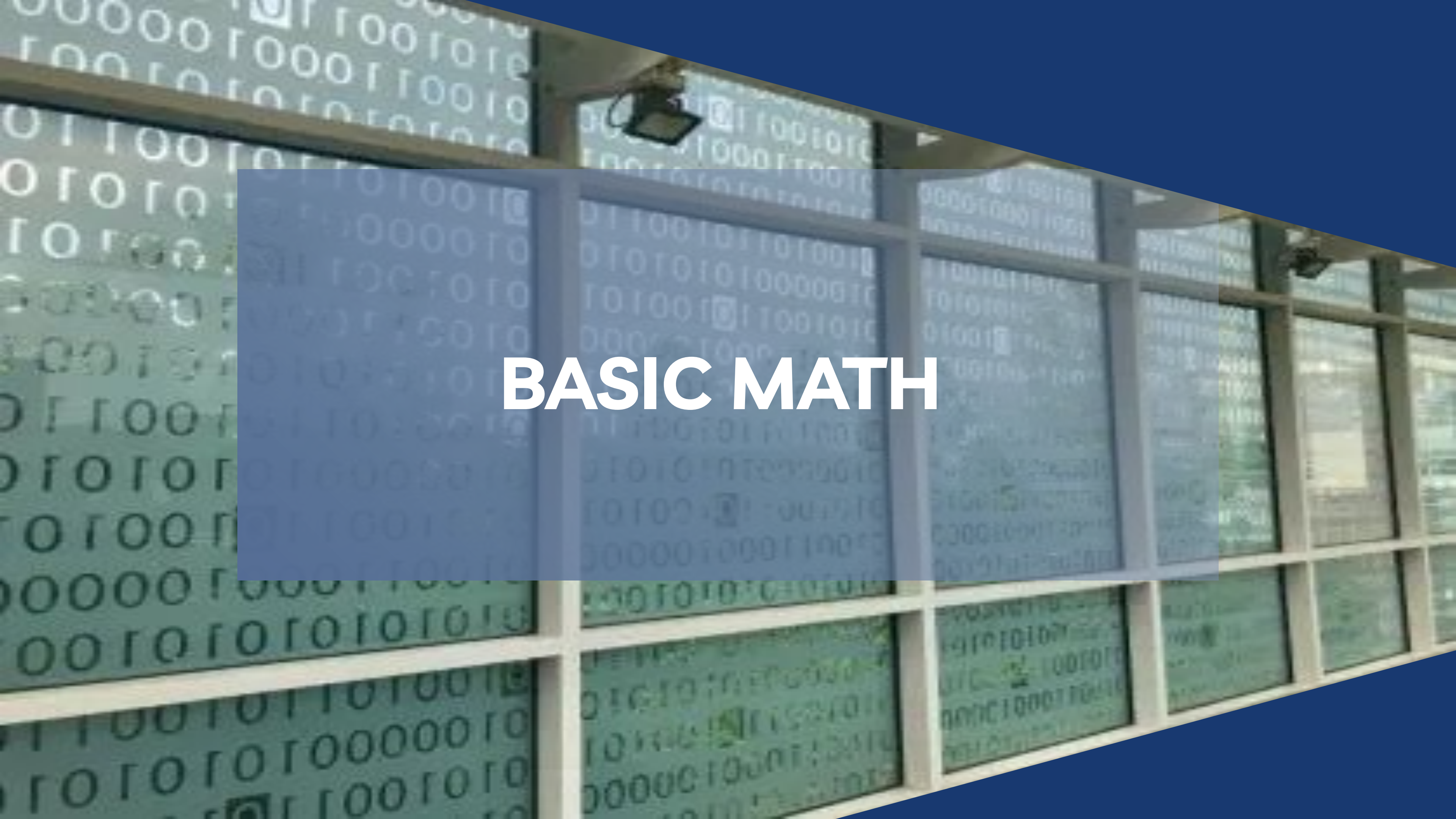
2. **EVENT PARAMETERS:**
   a. Each team may bring a collection of notes and resources, written/printed on paper, of any size containing information in any form and from any source. Binders, notebooks, folders, sheet protectors, lamination, tabs, and labels are permitted. Participants are responsible for organizing and containing their notes efficiently. They may separate or remove the pages from containers for use during any part of the event.
   b. Each team may also bring non-electronic tools and supplies, writing utensils, and two Class III calculators for use during any part of the event.
   c. Each team may bring one pre-constructed device.
   d. The Event Supervisors will provide the testing materials listed in the COMPETITION AREA section. Teams should not bring these materials.
   e. Participants must be able to answer questions regarding the design, construction, and operation of the device per the Building Policy found on www.soinc.org.

3. **CONSTRUCTION PARAMETERS:**
   a. The device must be a class 1 lever connected directly via a flexible or rigid link to a class 2 or 3 lever, each with a single "beam" of length that measures no longer than 40.0 cm. The "beam" is the bar that rests on the fulcrum and includes any attached components, except the flexible or rigid link between the two levers. Its length should be measured along the overall longest edge of the beam and is measured irrespective of the location of the test mass attachment points. It is measured without the supervisor-provided test masses attached.
   b. Springs, electric components, and electronic components are prohibited.
   c. The device must be constructed to accommodate the test masses as described in the COMPETITION AREA.
   d. Participants must not bring masses or include them in their device except when fixed in place prior to testing to obtain static equilibrium. Lightweight adjustable hooks that may be moved along the beam and are used solely to accommodate the test masses are allowed and need not be fixed in place.

4. **COMPETITION AREA:**
   a. The Event Supervisor will provide the testing materials listed below. The Event Supervisor must ensure that the mass measurements of the test masses and the ratios between those mass measurements are not revealed to any teams.
   b. Event Supervisors will supply three test masses labeled A, B, and C. A flexible loop, large enough to pass a standard golf ball through, must be tied to the top of each test mass. The loops may be made from fishing line, zip ties, string, etc. Each test mass, including the fully stretched out flexible loop, must be able to fit inside a 15.0 cm x 15.0 cm x 20.0 cm box. Each test mass, including the loop and container, must be between 20.0 g and 800.0 g. The ratio of the heaviest test mass to the lightest test mass must not exceed the following limits:

Regionals	States	Nationals
8.0	10.0	12.0

   c. The event supervisor may provide multiple testing stations, each with its own sets of test masses. The mass ratios between the test masses at each station must be identical to the precision that the Event Supervisor indicates in 5.II.g. The actual mass measurements of A, B, and C at each station must be roughly identical.
   d. An example where the Event Supervisor uses multiple stations and asks for a precision in the submitted ratios of one decimal point:
      i. Station 1: B1 has a mass of 300.0 g, A1 has a mass of 100.0 g. B1/A1 = 3.0
      ii. Allowed at Station 2:
         (1) B2 has a mass of 300.5, A2 has a mass of 99.8g.
         (2) B1/A1 = B2/A2 = 3.0 to the correct precision = Identical mass ratios. The actual masses are roughly identical. So this is allowed in 4.c.

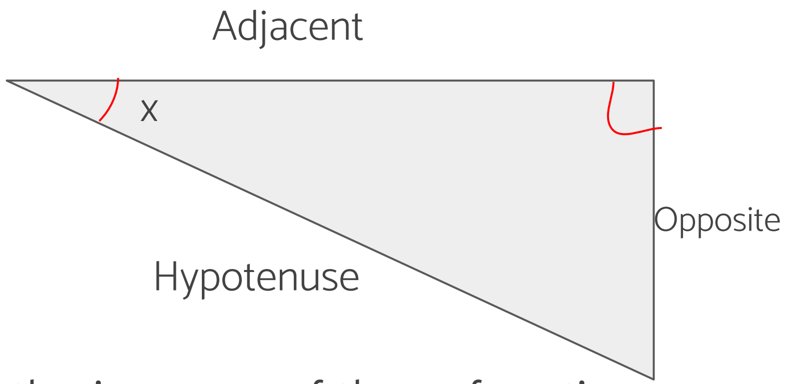


BASIC MATH

Basic Trig Crash Course

Picture a right triangle

- Angles add up to 180 degrees, or pi radians
- Let the angle x be the angle of interest
- Sin(x) = Opposite/Hypotenuse
- Cos(x) = Adjacent/Hypotenuse
- Tan(x) = Opposite/Adjacent

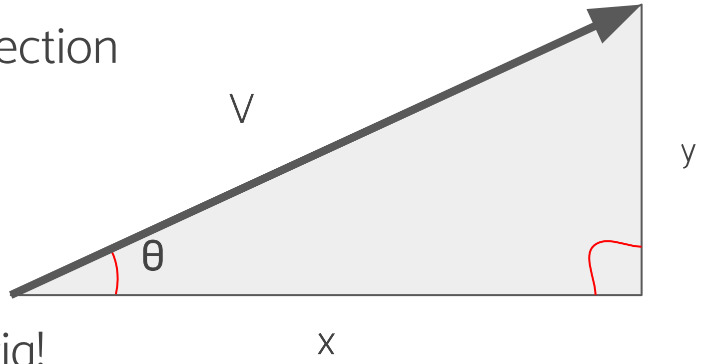


And similarly, arcsin, arccos, and arctan are the inverses of these functions

- arcsin(Opposite/Hypotenuse) = x
- arccos(Adjacent/Hypotenuse) = x
- arctan(Opposite/Adjacent) = x

The same can be done of the angle with the angle 90-x! Because the angle of interest changes, the adjacent and opposite sides flip and the same thing applies!

Applying Trig to Vectors

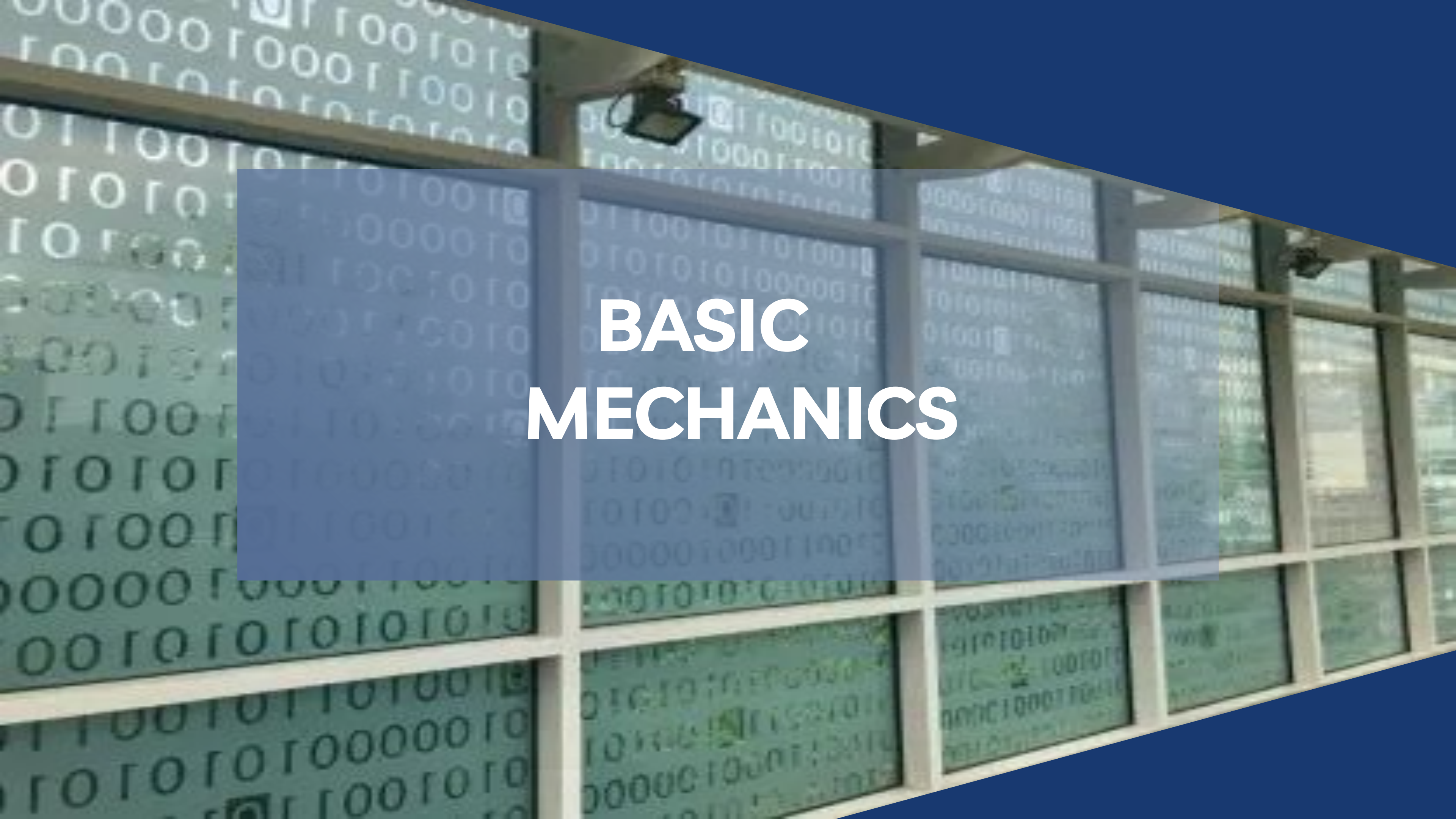


- A vector is a quantity with magnitude and direction
- This vector is in 2-D plane
- Can be represented in 2 ways:
  - Length and angle
  - x-component and y-component
- We can switch between the two forms with trig!
  - $\sin(\theta) = y/v \Rightarrow y = v^*\sin(\theta)$
  - $\cos(\theta) = x/v \Rightarrow x = v^*\cos(\theta)$
  - $\tan(\theta) = y/x \Rightarrow y = x^*\sin(\theta)$

And conversely...

  - V = sqrt(x^2 + y^2)   (Pythagorean Theorem)
  - $\tan(\theta) = y/x \Rightarrow \theta = \arctan(y/x)$ (Inverting the tan function)

Ta-dah!



BASIC MECHANICS

Kinematics!

- x is position, measured in [m]
- v is velocity(speed), the change in position [m/s]
- a is acceleration, the change in velocity over time [m/s^2]

Naturally, we have that $x = x_0 + vt$

What if velocity also changes? Then, we know that $v = v_0 + at$

Using (calculus), we can determine that $x = x_0 + v_0 t + \frac{1}{2}at^2$

It can also be determined that $v^2 = v_0^2 + 2a(x - x_0)$

You can apply this to any object in a simple system, and approximate complicated systems really well!

→ these equations allow for solving of position, velocity, and acceleration for any moment in time during motion of an object.

Forces & System Equilibrium

$$F = ma$$

- A force is something that changes the system
- It is measured in Newtons, or [kg*m/s^2]
- Changes the speed of something -> affects acceleration
- Forces add up! Hence we can calculate a total force (and total acceleration

$$\sum F = ma$$

$$\sum F_x = ma_x$$

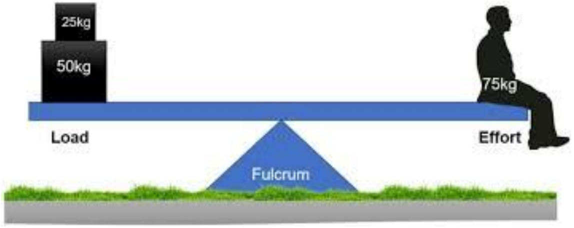
$$\sum F_y = ma_y$$

- When the system does not move, this means that $\sum F = 0$ and so $\sum F_x = 0 \quad \text{and} \quad \sum F_y =$

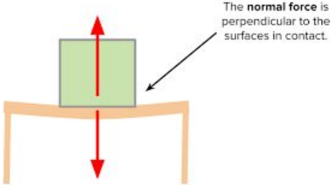
(When adding forces(vectors), break them down into their xy components, add them in their respective dimensions, and transform back to overall vector form)

- The inverse is true
- Allows us to solve static systems
- Prime example of a force:

$$F_g = mg$$

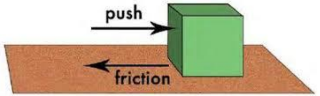


Friction



- Normal force: an inherent "invisible" force which allows for the system to work out
  - In problems, usually comes from hardness of a material or gravity
  - Ex: pushing down on a table

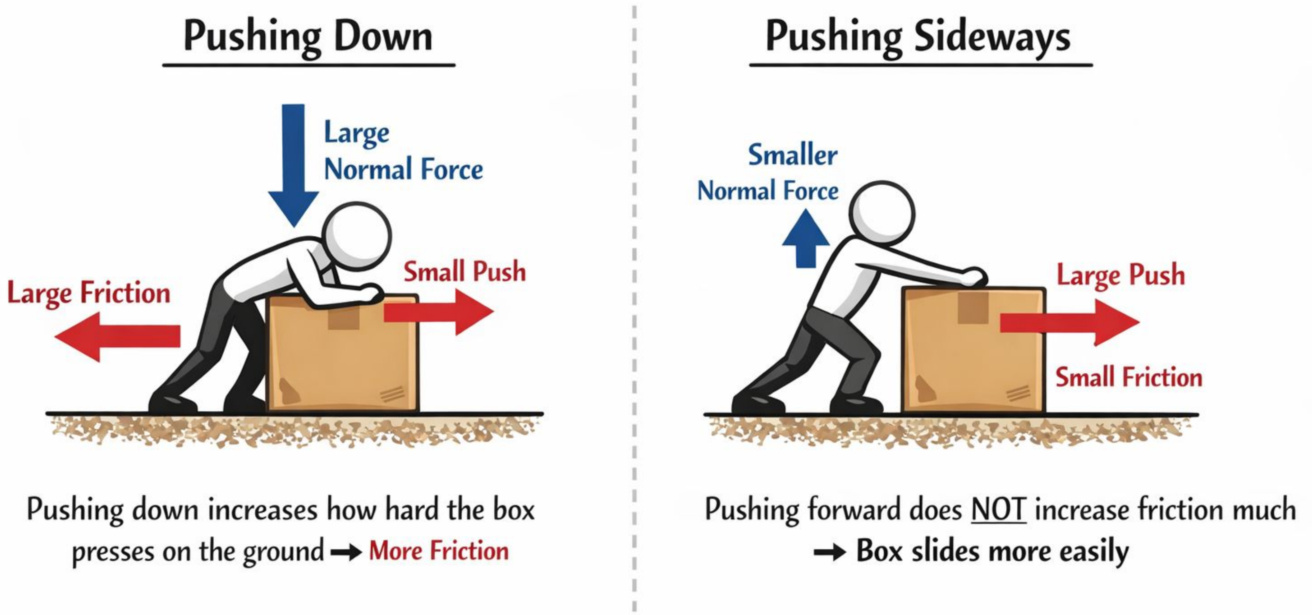
- Friction is a non-conservative force → loss of efficiency in the system



  - E.g. think of dragging a wooden box on a rough surface
  - Friction opposes the direction of motion → makes it harder to move!

- Friction is a multiple of this "Normal Force": $F_f = \mu N$
  - Can you figure out why?
- For a simple object sitting on a surface, the normal force $N = mg$

Friction



$$F_{friction} = \mu N$$

Static vs Kinetic

- Static Friction is occurring when the object on which friction is being exerted is currently at rest
- Kinetic Friction is occurring when the object on which friction is being exerted is currently moving
- The coefficients of static friction and kinetic friction are different!
  - Harder when you start to move something than to keep moving it
  - Coeff. of static friction is always >= coeff of kinetic friction

$F_f = \mu N$ $F_{s,\max} = \mu_s N$ $F_k = \mu_k N$ $\sum F = F_{\text{applied}} - F_f$

Recap of Kinematic Equations

Kinematics	Forces & Eq.	Friction

Kinematics

$$x = x_0 + vt$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Forces & Eq.

$$F = ma$$

$$\sum F = ma$$

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$F_g = mg$$

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

$$\sum F = 0$$

Friction

$$F_f = \mu N$$

$$F_k = \mu_k N$$

$$F_{s,\text{max}} = \mu_s N$$

$$\sum F = F_{\text{applied}} - F_f$$

WORK AND ENERGY

Work & Energy

- Work is defined as the use of energy to apply a force over a distance
  - W = F*d, or, more formally, $W = \int \mathbf{F} \cdot d\mathbf{s}$ which often gives $W = Fd\cos(\theta)$
  - Energy put into the system
- Energy is defined as the ability to do work, two different kinds
  - Kinetic Energy (energy of movement); $K = \frac{1}{2}mv^2$
  - Potential Energy (energy of position);
    - For gravity, $U = mgh$
    - For springs, $U = \frac{1}{2}kx^2$
    - For electric charges, $U = \frac{kq_1q_2}{r}$
  - Notice how KE is the integral of momentum, while U is integral of Force!
- Circular Definition!

$W = \Delta E$ $W = \Delta U + \Delta KE$

Rotational Energy

- Same exact concepts for rotations!!
- But what is "mass" in rotation?
  - Moment of inertia – How "hard" it is to rotate something – $I = \int_m r^2 dm$
  - Put a bunch of these in your binder!!! No need for calculus.
- Redefining Forces and Energy in Rotational Kinematics,
  - A rotational force is called a Torque, $\tau = I\alpha = \mathbf{r} \times \mathbf{F} = rF \sin\theta$
  - Rotational Kinetic energy is $KE_{\text{rot}} = \frac{1}{2}I\omega^2$
  - Potential Energy is the same, there is no such thing as rotational potential
- Relationship between linear and rotational lies in difference between speeds and accelerations $a = \alpha r$ $v = \omega r$
  - A ratio of the radius of rotation! $E = KE_{\text{trans}} + KE_{\text{rot}} + U$
- Switching between the two can help solving complex systems

Rotational mechanics

Translational	Rotational
$F = ma$	$\tau = I\alpha$ (= Fr)
$v = v_0 + a\,t$	$\omega = \omega_0 + \alpha\,t$
$\Delta x = \frac{1}{2}\,(v_0 + v)\,t$	$\Delta\theta = \frac{1}{2}\,(\omega_0 + \omega)t$
$\Delta x = v_0 t + \frac{1}{2}\,a\,t^2$	$\Delta\theta = \omega_0\,t + \frac{1}{2}\,\alpha\,t^2$
$v^2 = v_0^2 + 2\,a\,\Delta x$	$\omega^2 = \omega_0^2 + 2\,\alpha\,\Delta x$

Conservation of Energy

Conservation of energy – for a closed system with only conservative forces, all of the energy is preserved at any moment in time no matter what happens in the system

- Example: ball running down a hill, jumping, etc

Formally, $E_i = E_f$ and so $KE_i + U_i = KE_f + U_f$

Conservative force: path-independent

- Energy in system remains unchanged

Non-conservative force: path-dependent E.g. friction

For an open system with external and non-conservative forces

$$K_i + U_i + W_{\text{nc}} = K_f + U_f$$

So what is a system?

- Anything which is relevant in the given scenario or which contributes to a change in energy of any component, including itself. E.g. a group of objects

SIMPLE MACHINES

Simple machines

- **Simple machines can...**
    - Change direction of force
    - Reduce the force needed to do the same amount of work
    - Reduce the distance needed to do the same amount of work
- Recall Work = Force * distance

Idea: amount of work done remains the same

$$W = F_{in} d_{in} = F_{out} d_{out}$$

Simple machines overview

Relevant machines

1. First, second, and third class levers
2. Inclined planes
3. Wedges
4. Wheel and axle (and gears)
5. Pulleys
6. Screws

Calculations

- Solving for mechanical advantage
- Solving for efficiency
- Solving for work done

TL;DR – Using basic mechanics, solve for different parts of the equations.

Why care?

- **Simple machines can...**
  - Change direction of force
  - Reduce the force needed to do the same amount of work
  - Reduce the distance needed to do the same amount of work
- Recall Work = Force * distance

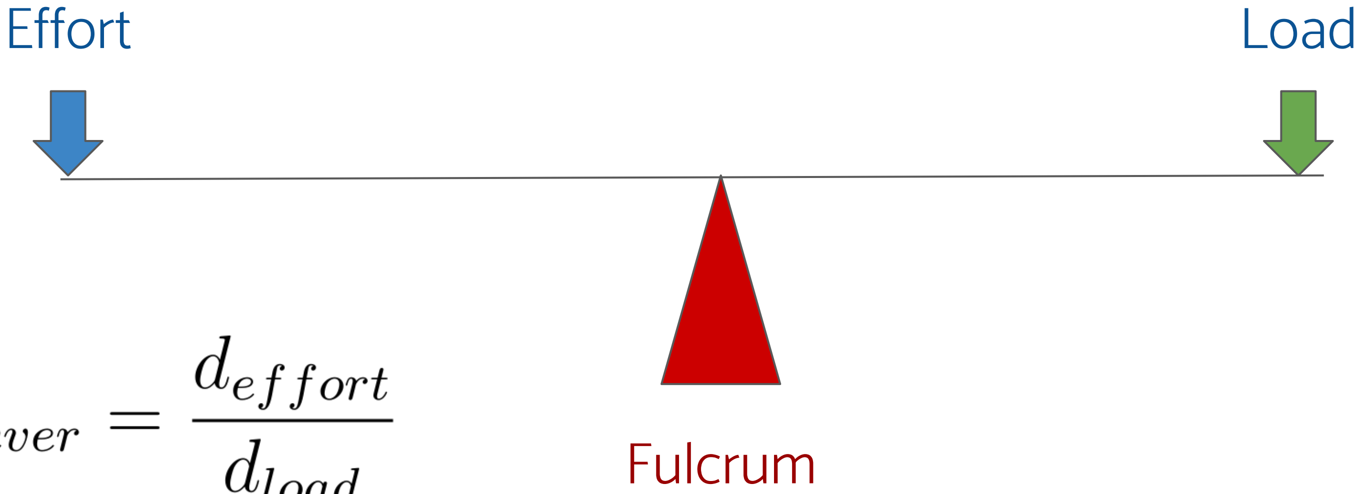
Ideal mechanical advantage

Idea: amount of work done remains the same

$$IMA = \frac{F_{out}}{F_{in}} = \frac{d_{in}}{d_{out}}$$

Levers

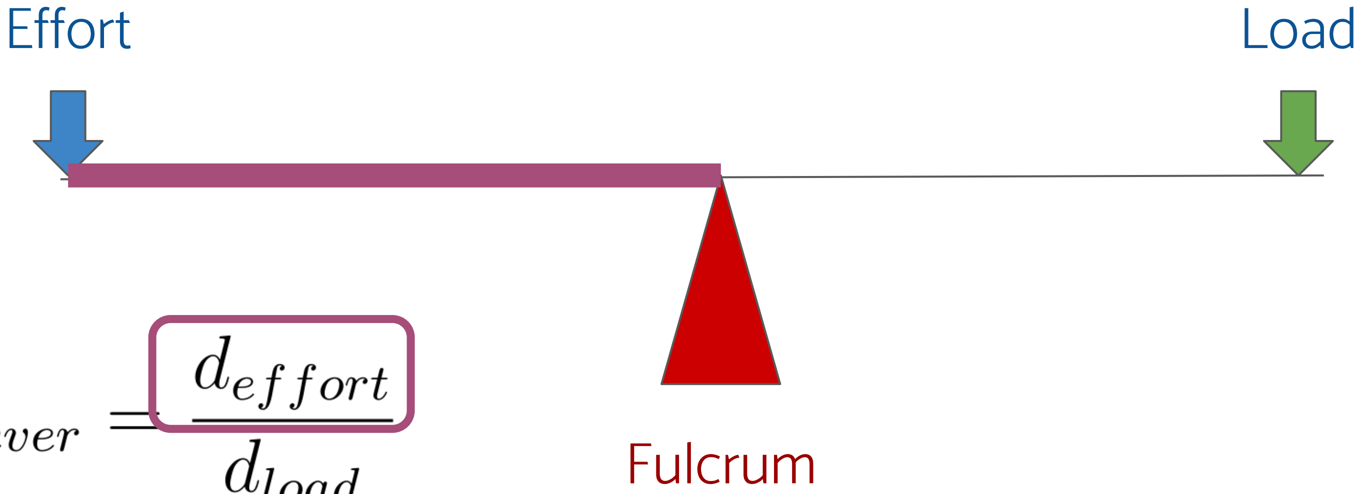
Class 1 Lever



$$IMA_{lever} = \frac{d_{effort}}{d_{load}}$$

Levers

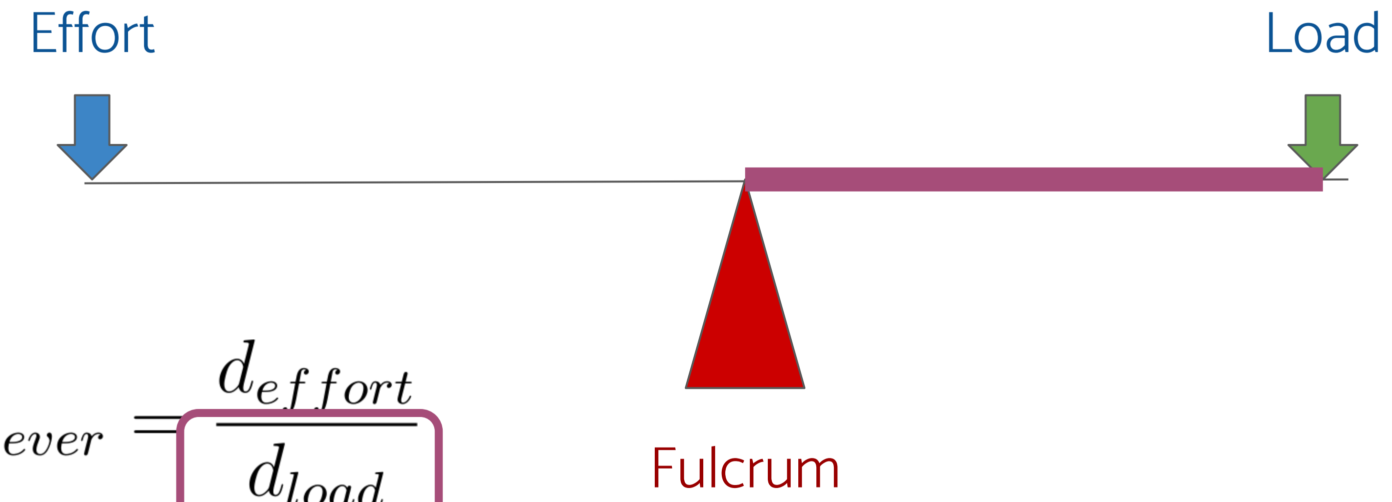
Class 1 Lever



$$IMA_{lever} = \frac{d_{effort}}{d_{load}}$$

Levers

Class 1 Lever

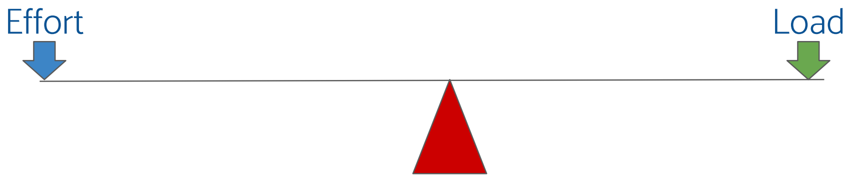


$$IMA_{lever} = \frac{d_{effort}}{d_{load}}$$

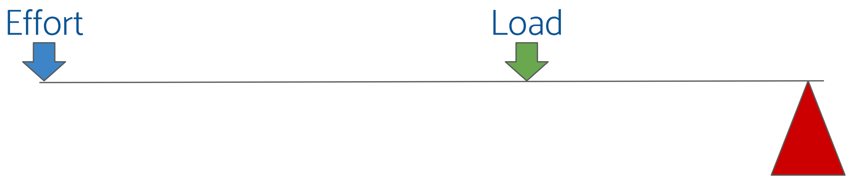
Levers

$$IMA_{lever} = \frac{d_{effort}}{d_{load}}$$

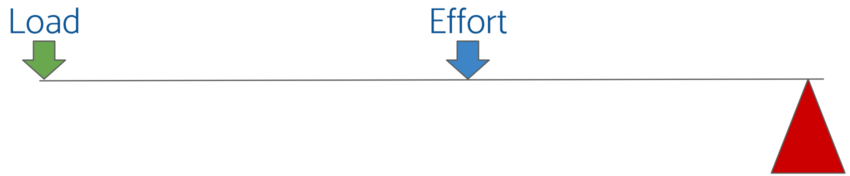
Class 1 Lever



Class 2 Lever

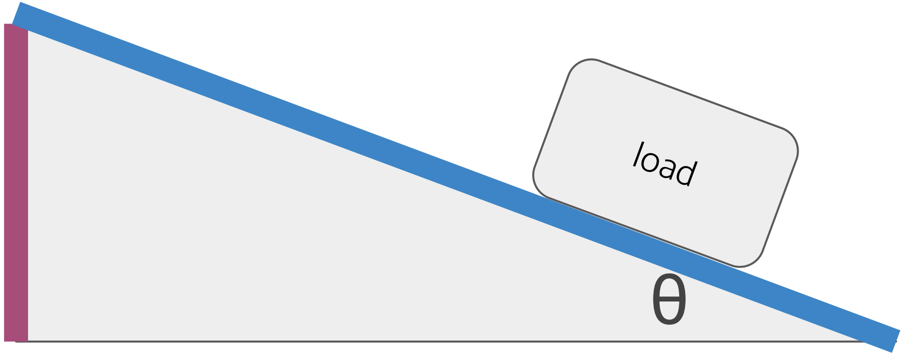


Class 3 Lever



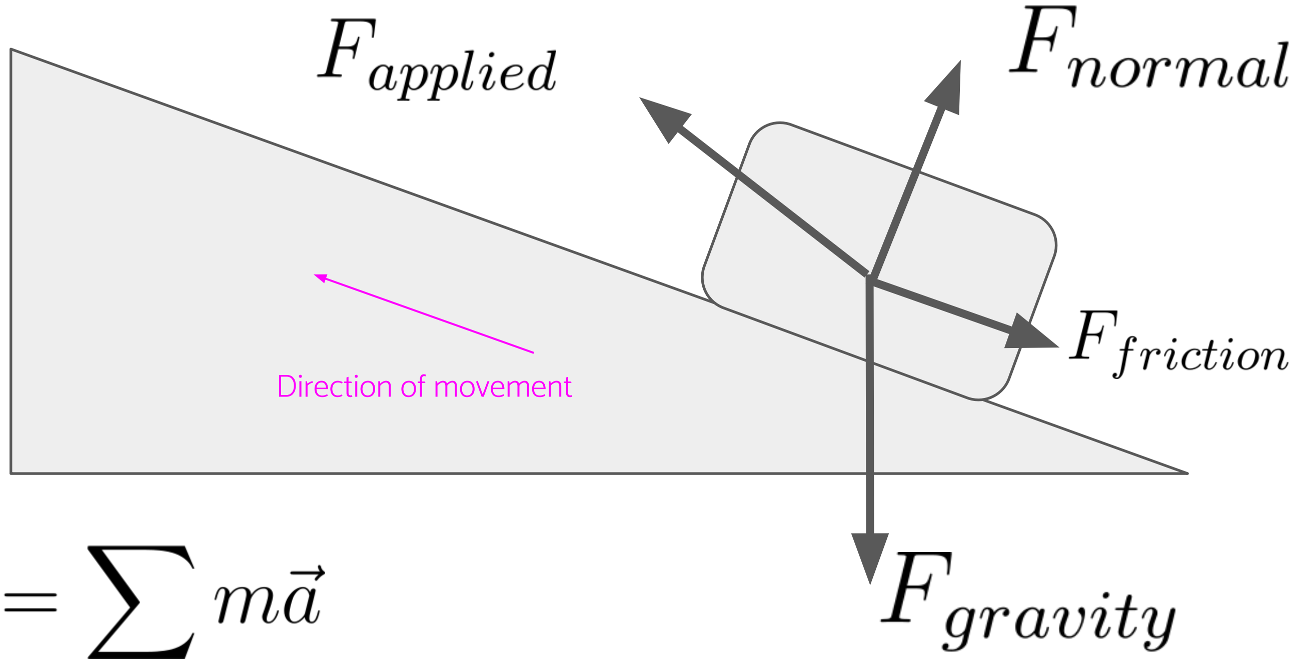
Inclined Planes

$$IMA_{\text{inclined plane}} = \frac{d_i}{d_v}$$



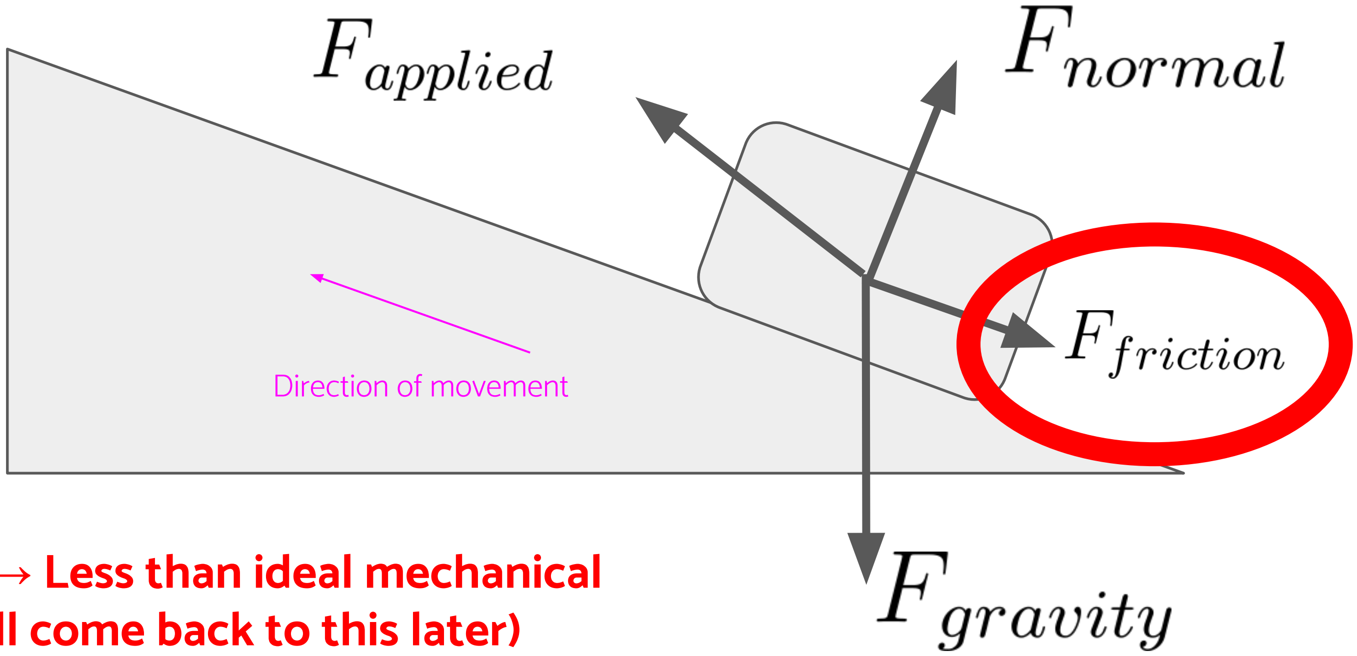
$$d_i = \cos(\theta) d_v$$

Setting up forces on inclined planes



Problem setup: $\vec{F} = \sum m\vec{a}$

Setting up forces on inclined planes

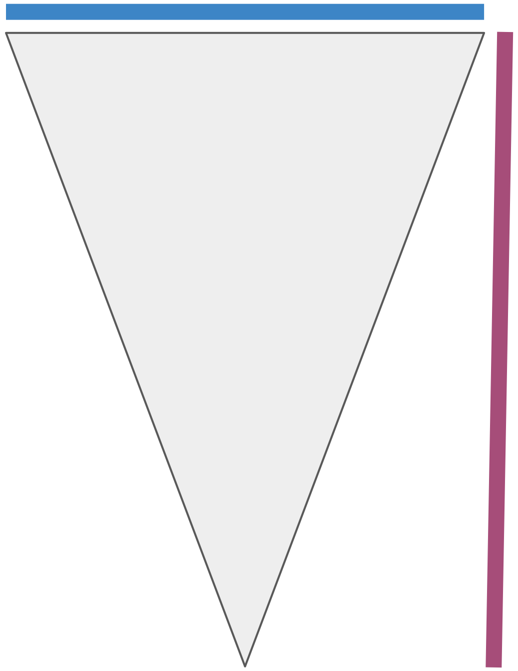


|F_friction| > 0 → Less than ideal mechanical advantage (we'll come back to this later)

Wedges

Examples: axe head, knives, doorstops, zippers

$$IMA_{wedge} = \frac{L}{w}$$

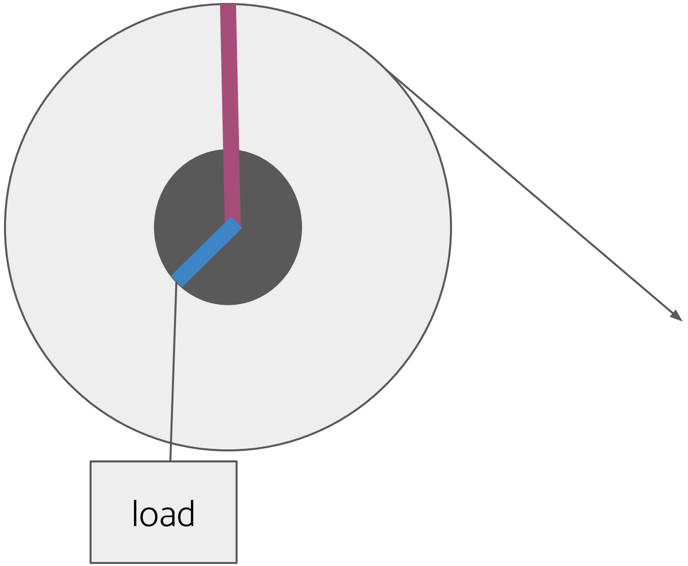


Wheels and Axles

- For a simple wheel and axle
- Note the load and effort can be reversed (invert given IMA equation)

Problem set-up: using ratios

$$IMA_{wheel} = \frac{R}{r}$$

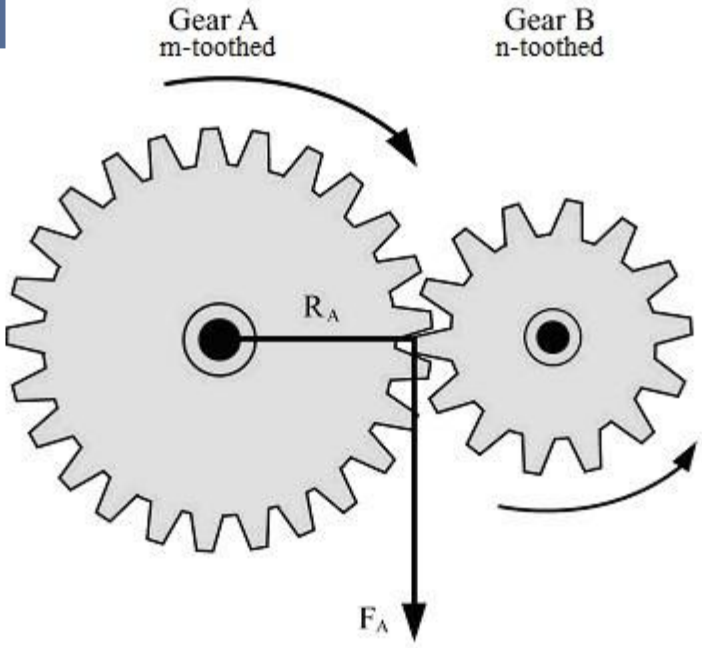


Gears

- **Type of compound machine**
- Gear systems
  - Transmission of torque
  - Problem set-up:
    - Use gear tear count to define ratios
    - Use ratios to solve input/output force or distance problems

Intuition

↑gear A teeth ⇒ ↑ number of rotations



Gear B rotates faster than gear A by m/n.

Image from the Scioly Wiki.

Pulleys

- Single pulley: class 1 lever
- More useful with multiple pulleys

- **Heuristic**:

  IMA = count the number of supporting lines lifting the load

$$IMA_{pulley} = \frac{d_{in}}{d_{out}} = \frac{F_{out}}{F_{in}}$$

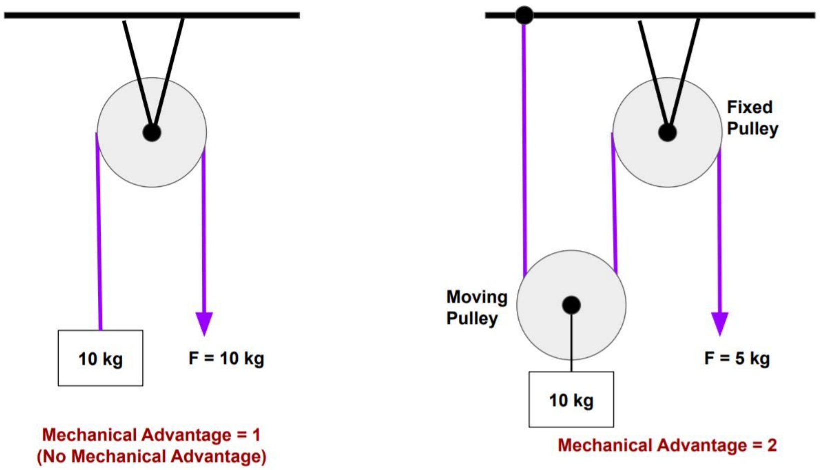


Image source

Screws

Inclined plane wrapped around a cylinder

- P := pitch (distance between threads)
- L := "radius" of the turn of the screw

Recall: circumference of circle is 2πr

$$IMA_{screw} = \frac{2\pi L}{P}$$

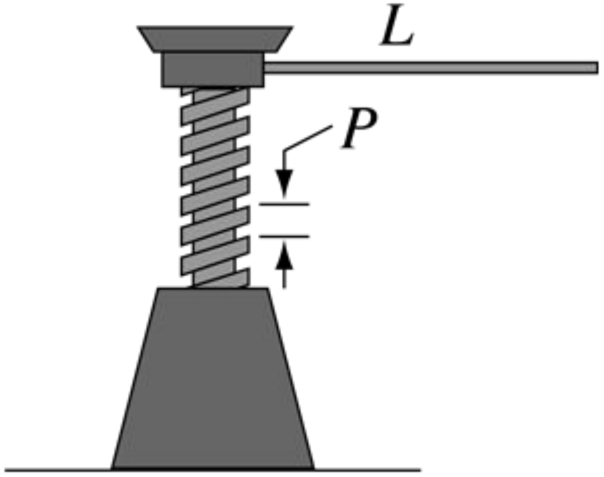


Image from Hyperphysics.

Example Generic Problem Types

- Calculate mechanical advantage
  - Comparing ideal MA to actual MA (with friction)
- Calculate net force
  - Calculate **acceleration**

Q: How does acceleration work with spinning objects?

Rotational Mechanics

Question: How do kinematics relate to simple machines?

- Special case for rotational kinematics
  - Describes spinning motion, not straight line movement
- Motion
  - Angle: θ
  - Angular velocity: ω
  - Angular acceleration: α
- Forces
  - Torque: τ

Relationship to translational kinematics:

$v = r\omega$ $\Delta s = r\,\Delta\theta$ $a = r\alpha$

Rotational mechanics

Translational	Rotational
$F = ma$	$\tau = I\alpha$ (= Fr)
$v = v_0 + a\,t$	$\omega = \omega_0 + \alpha\,t$
$\Delta x = ½\,(v_0 + v)\,t$	$\Delta\theta = ½\,(\omega_0 + \omega)t$
$\Delta x = v_0 t + ½\,a\,t^2$	$\Delta\theta = \omega_0\,t + ½\,\alpha\,t^2$
$v^2 = v_0^2 + 2\,a\,\Delta x$	$\omega^2 = \omega_0^2 + 2\,\alpha\,\Delta x$

What is I?

- **I := moment of Inertia**
  - “Mass” analogue in rotational kinematics

Relevant formulas:

Cylinder or disk, symmetry axis - Pulleys - Wheel and axle	$I = \frac{1}{2} MR^2$, R := radius
Rod around non-center - Lever	$I = \frac{1}{3} ML^2$, L := length
Point mass	$I = Mr^2$, r := distance from axis of rotation

Ideal versus actual MA

Recall: friction acts **against** direction of motion – loss of energy

AMA := actual mechanical advantage
IMA := ideal mechanical advantage

$$\eta = \frac{F_{out}}{F_{in}} = \frac{AMA}{IMA}$$

Example problem types:

- Given the efficiency, calculate the actual force needed to do *X* amount of work
- Given the output force and some input force, calculate the efficiency
- Given the coefficient of friction, calculate the efficiency of the system

Additional Resources

Rules Manuals

Division B

Division C

Machines Wiki

Science Olympiad Test Archive

Kinematics Examples and Math

THANKS!